**Design Project #2: Kinematics of a Linkage System**

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**Introduction**

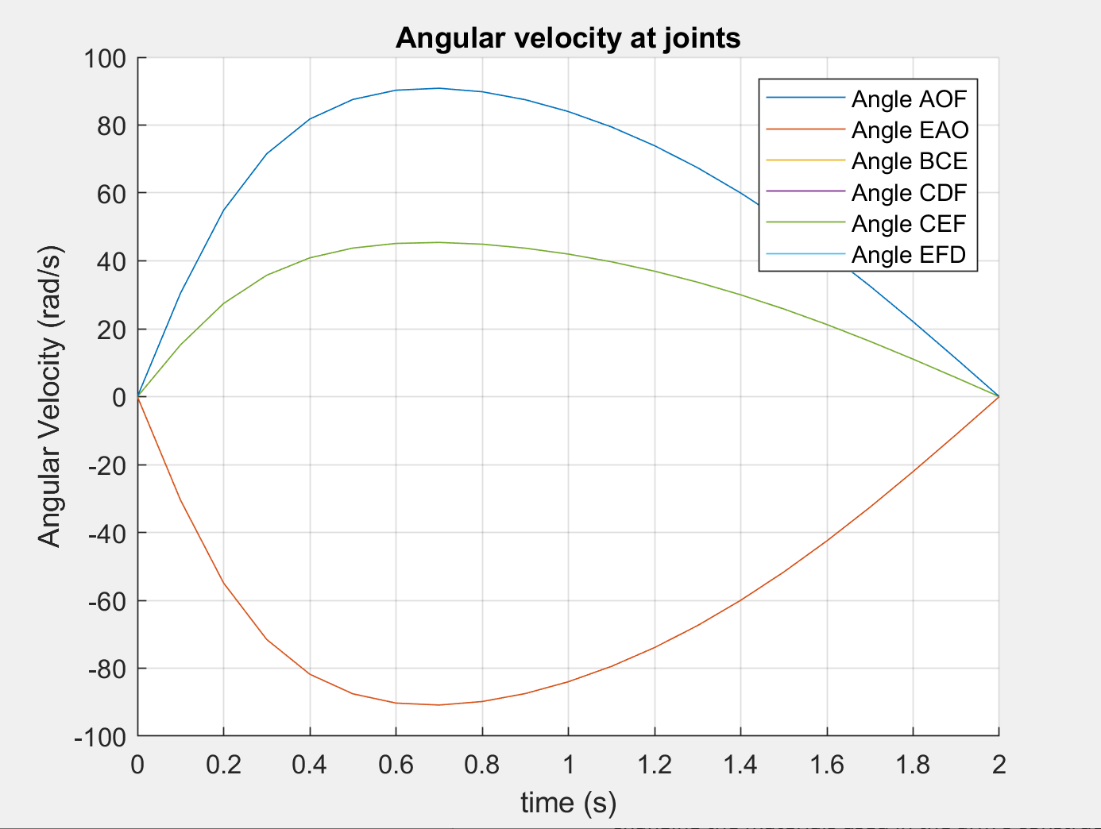
The second design project had us perform an analysis of the linkage system of an umbrella’s arm [DAN 1]. Using concepts learned in lecture sections and the software MATLAB, linkage kinematics and position dynamics was used in order to calculate the displacements, velocities, and accelerations of the components of the mechanism. The system analyzed contained six links, which was different from the four-bar linkage systems we solved during lecture and in the homework. Thus, needing to scale up our analysis on MATLAB in order to accommodate the increase in the number of variables needed to be solved was necessary. This experiment is important because as a result of conducting this analysis the mechanism can be broken up and numerical data on how each linkage in the system behaves when the arm is extended and contracted can be obtained. It is predicted that approximately between 1640 and 25,900 umbrella-related incidents that result in injury occur globally each year [DAN 2]. These incidents can lead to the onset of mass recalls and lawsuits which can drive a manufacturer out of business. Also in the scenario of mass production, material costs heavily affect the profit margins of each umbrella sold. If our team was a manufacturer of umbrellas for instance, this information can give useful suggestions on how to alter the mechanism design so that the umbrella is more user-friendly, safe to use, and affordable to the customer.

**Analytical Procedure**

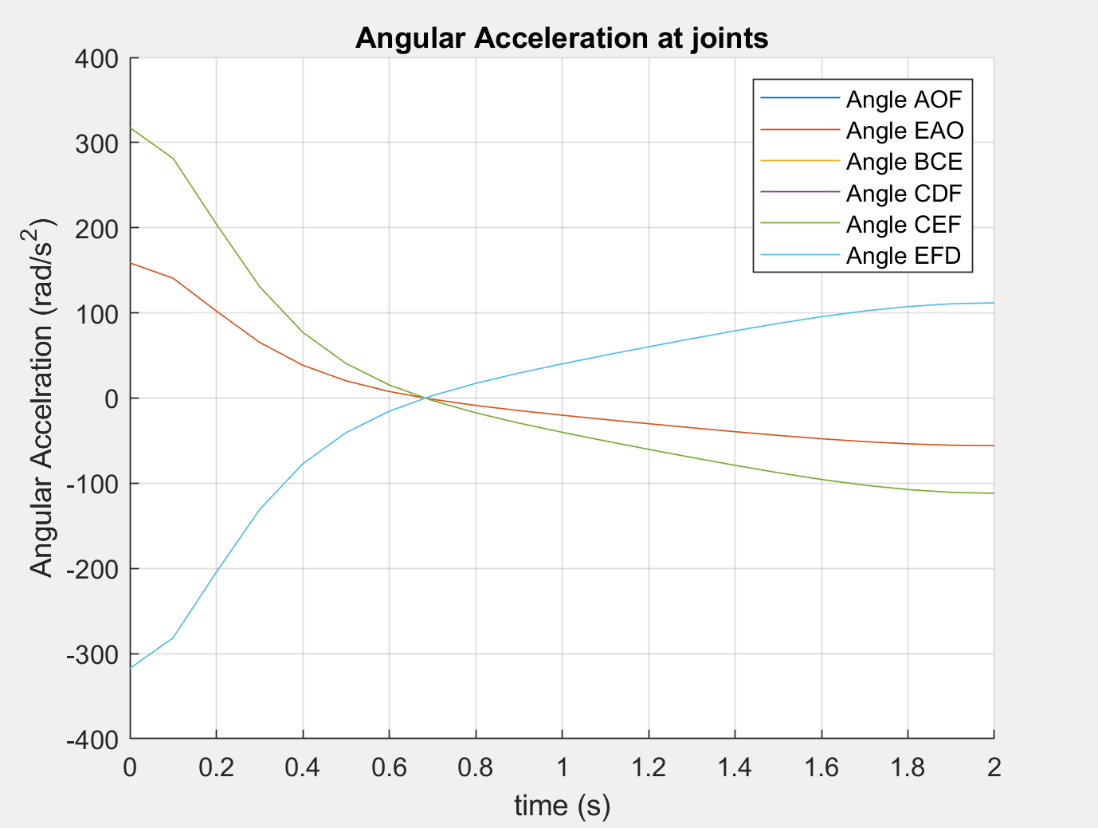
Using the given parameters, the team was able to formulate mathematical method to calculate the angles at of the linked elements. With the provided lengths of bars, using the Law of Sines and Cosines enabled the team to find the angles of angle AOF, angle EAO, angle BCE, angle CDF, angle CEF, and angle EFD of the initial orientation at time . When calculating for the angles in quadrangle CDEF, an arbitrary line was calculated to divide it into two triangles CDF and CEF to calculate angle CFD and angle CFE, and then combined to find angle EFD. Angle CDF and CEF was found using trigonometric identities from triangle AOE, and angle DCE is found because of the internal angles of quadrangle being 360 degrees. By setting the length OA as a function of time as provided: , the forementioned angles can be written in terms of and , and as a result, functions of time. Once the angles were discovered, using the *diff* function in MATLAB allowed the team to differentiate the angles in terms of to find angular velocities and accelerations. Further utilizing the angles written as a function of time, trigonometry -mainly law of sines and cosines- allowed the team to find the position of points A, B, C, D, E, and F on - plane in terms of function of time. The opening time was given as , and t has to be within . Array of in increment of from 0 to 2 was substituted into the equations to plot the trajectory of points A, B, C, D, E, and F. Furthermore, the equations of these points on - plane was differentiated with respect to to derive and plot linear velocities and linear accelerations in x and y directions.

The MATLAB code is able to plot angles, angular velocities, and angular accelerations of angle AOF, angle EAO, angle BCE, angle CDF, angle CEF, and angle EFD versus time, and trajectory, linear velocities, and linear accelerations of points A, B, C, D, E, and F on - plane. Additionally, the code can execute the calculations with different provided lengths of , , , , , , and , as long as the configuration of the elements are unchanged.

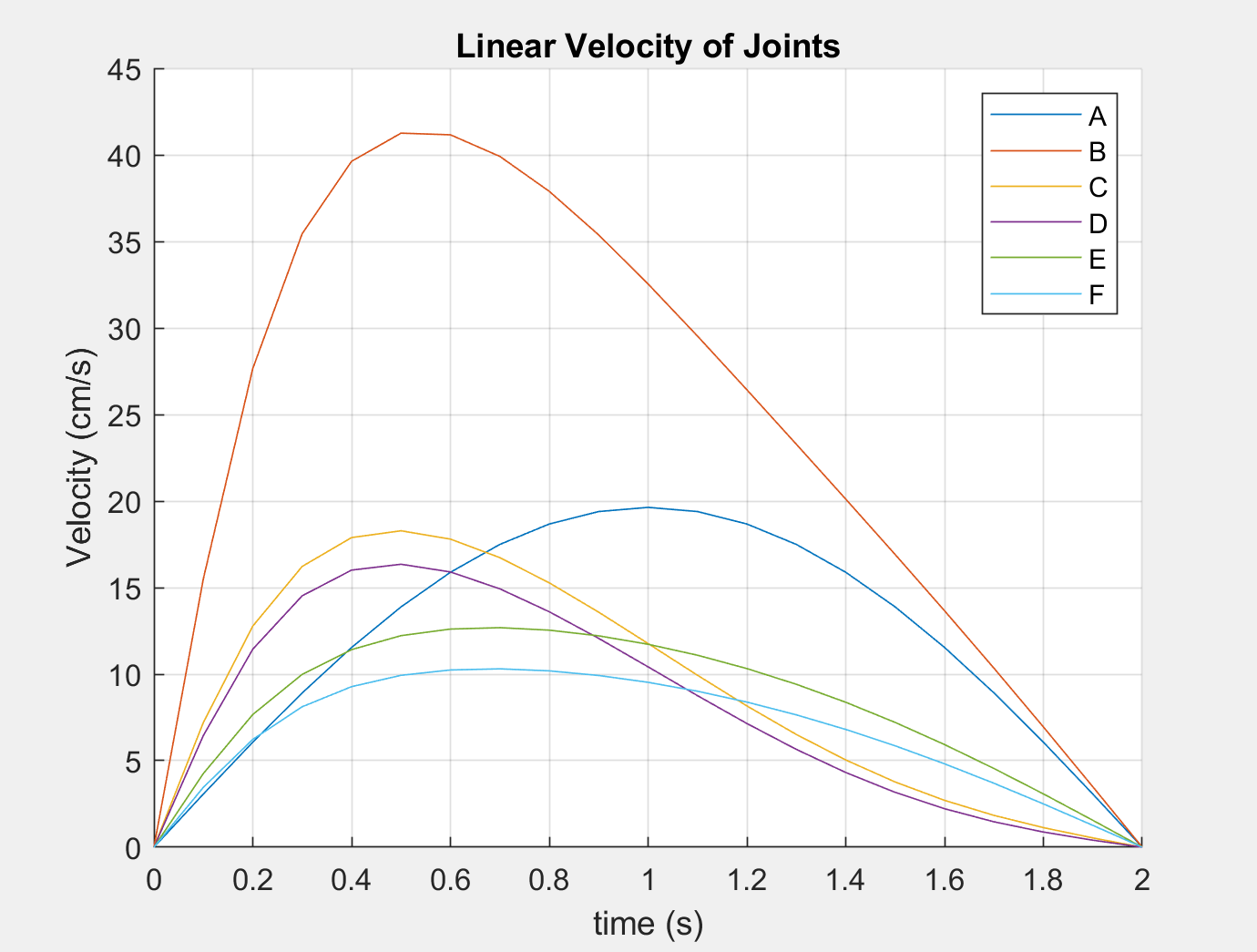
**Figure 1:**



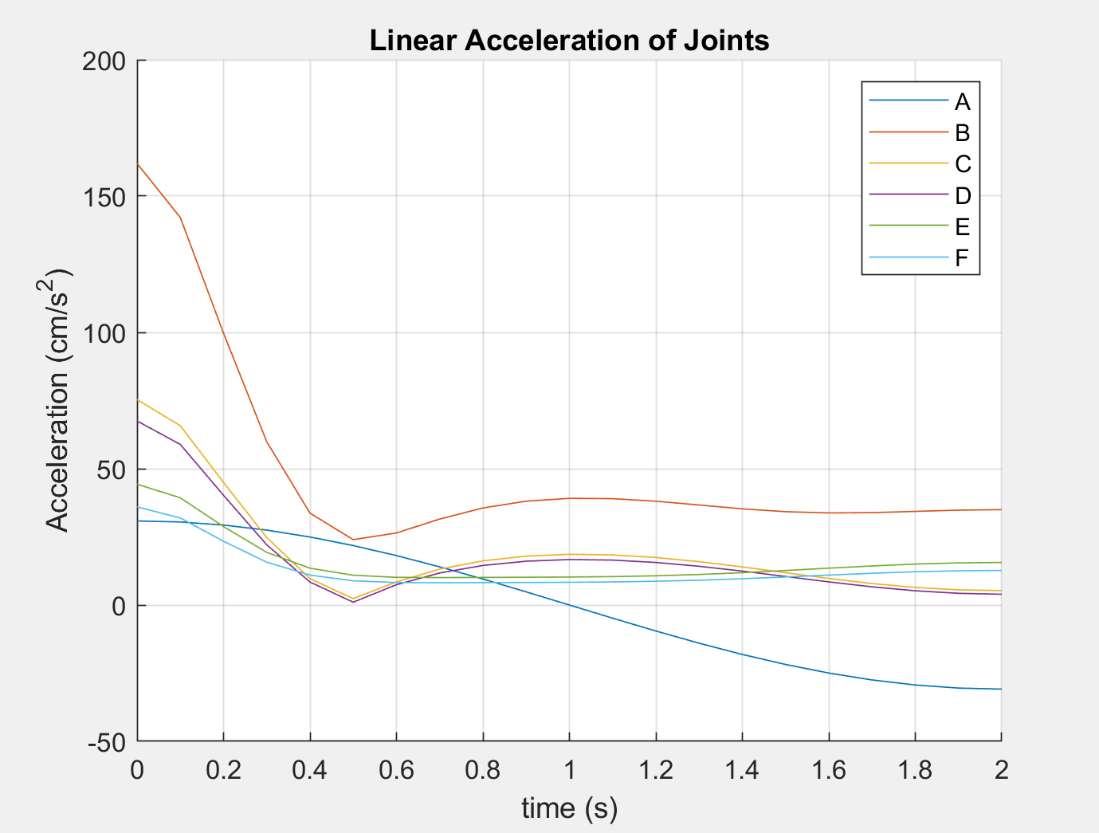
**Figure 2:**



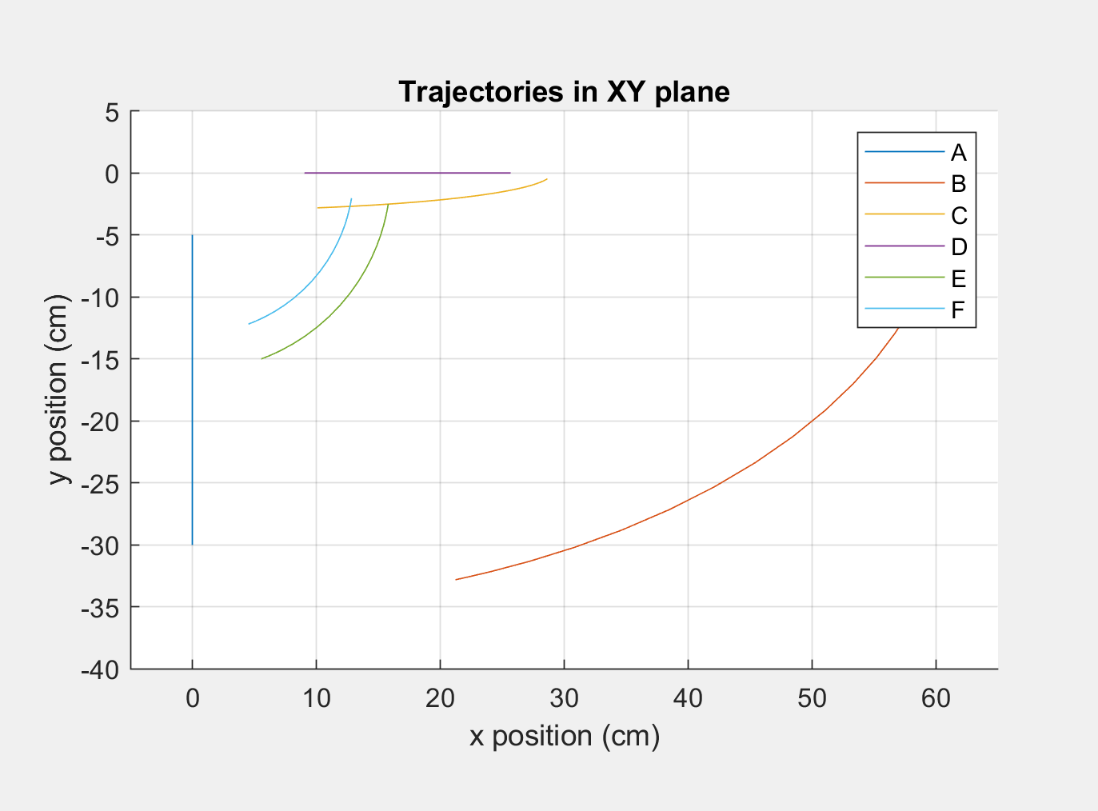
**Figure 3:**



**Figure 4:**



**Figure 5:**



**Interpretation**

**Descriptions of the Significant Results in the Plots**

In Figure 1, the angular speeds of each linkage in the mechanism reaches its peak at around 0.7 seconds. Member CDF and CEF had an equal angular speed of 90.85 rad/sec. In Figure 2, the angular accelerations of the linkages all intersect when the time is around 0.684 seconds, confirming the observations made in Figure 1. The angular accelerations crossing each other at (0.684, 0) states that at t=0.684, each linkage reaches its maximum angular speed and reverses direction. Figure 3 shows the linear velocity of the joints of the system, this is different from Figure 1 in the fact that the peak velocities for the joints do not line up when t= 0.684. Similarly, to Figure 1, all linkage velocities start at 0 and return to 0 when t= 2 seconds. This states that the experiment constitutes one complete motion before ending at full extension. This is also represented in Figure 4 as well, however unlike Figure 3 which represented the angular accelerations, the linear accelerations of the members don’t all intersect at a specific time at 0. This shows that the members of the system do not reverse the direction of their linear motion as the arm of the umbrella extends.

Figure 1 displays the angular velocities at the joints of the umbrella during the time interval of 2 seconds. Angles EFD and CEF share the highest magnitude of angular velocity in the system. Angle EFD has negative values while angle CEF has positive values. This suggests that as angle CEF opens, angle EFD closes at the same angular velocity. The mentioned angles reach a peak velocity of 45.43 rad/s at t=0.7s. These results are reflected in Figure 2 (Angular Acceleration at Joints) as well. Angles EFD and CEF have equal magnitudes of angular acceleration yet they are in opposing directions for any time ‘t’ between 0 and 2 seconds. Figures 3 and 4 display the linear velocity and accelerations for all joints for every ‘t’ between 0 and 2 seconds. These graphs show that Joint B maintains the highest linear velocity and acceleration during its travel. Joint B’s peak velocity through 2 seconds is 4.24 cm/s at t=0.5 s. Joint B reaches peak acceleration upon opening (t=0 s) at 36.01 cm/s^2. Figure 5 displays a graph of the trajectories of all the joints in the XY plane. The graph shows that Joint B travels the largest distance, while joint F covers the least amount of distance. Joint D solely moves horizontally whereas Joint A solely moves vertically. The graphs all show that, during the 2 second opening interval, Joint B is the most dynamic.

**Engineering Observations About Utilization of Plots**

These plots visually represent the numerical data obtained as the arm of an umbrella extends as the canopy is unfurled. The plots our team generated outline the motion of each linkage of the assembly as the links rotate radially and move outward in a linear fashion. With this data our team can understand and display how quickly the umbrella arm extends. Also, with this information we can make educated predictions as to which components of the umbrella mechanism will experience more wear and tear during everyday use and redesign those components to make the system more durable by changing the materials used in the arm’s construction or the geometry of the linkages. Finally, in the scenario of mass production, by using the information obtained we can make more cost-efficient material selections that will lower the cost of production and thus increase the profit margin of each umbrella sold.

**Appendix**

See code attached with project submission.

**Works Cited**

[DAN 1] Blum, Michelle. *Design Project #2 Kinematics of a Linkage System*. Syracuse University, 21 Jan. 2021, learn-us-east-1-prod-fleet01-xythos.content.blackboardcdn.com/blackboard.learn.xythos.prod/5956621d575cd/13951762?X-Blackboard-Expiration=1615604400000&X-Blackboard-Signature=SzeP17UeDjiJfGmZoGF%2FrKtCwLHRl6tNsZDvYQcgfgM%3D&X-Blackboard-Client-Id=311690&response-cache-control=private%2C%20max-age%3D21600&response-content-disposition=inline%3B%20filename%2A%3DUTF-8%27%27MEE332\_Project\_2\_SP21%25281%2529.pdf&response-content-type=application%2Fpdf&X-Amz-Algorithm=AWS4-HMAC-SHA256&X-Amz-Date=20210312T210000Z&X-Amz-SignedHeaders=host&X-Amz-Expires=21600&X-Amz-Credential=AKIAYDKQORRYTKBSBE4S%2F20210312%2Fus-east-1%2Fs3%2Faws4\_request&X-Amz-Signature=2027e524ff4dc0742d13782fbccc4e82cf2cdcfa2010e8c227b43009d2bd7c5c.

[DAN 2] “How Many People Are Taken to Hospitals Every Year Due to Umbrella-Related Injuries?” *Quora*, [www.quora.com/How-many-people-are-taken-to-hospitals-every-year-due-to-umbrella-related-injuries](http://www.quora.com/How-many-people-are-taken-to-hospitals-every-year-due-to-umbrella-related-injuries).

[DAN 3] “Search.” *CPSC.gov*, 18 Sept. 2020, [www.cpsc.gov/search?site=cpsc\_site&output=xml\_no\_dtd&getfields=%2A&tlen=120&client=ek\_drupal\_01&proxystylesheet=ek\_drupal\_01&filter=p&query=umbrella](http://www.cpsc.gov/search?site=cpsc_site&output=xml_no_dtd&getfields=%2A&tlen=120&client=ek_drupal_01&proxystylesheet=ek_drupal_01&filter=p&query=umbrella).